INDEFINITE INTEGRATION

Integration is the inverse process of differentiation. That is, the process of finding a function, whose differential coefficient is known, is called integration.

If the differential coefficient of F(x) is f (x), i.e. $\frac{d}{dx}[F(x)] = f(x)$, then we say that the **antiderivative** or **integral** of $f(x)$ is F(x), written as $\int f(x)dx = F(x)$,

Here $\int dx$ is the notation of integration f (x) is the integrand, x is the variable of integration and dx denotes the integration with respect to x.

1. INDEFINITE INTEGRAL

We know that if $\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x)dx = F(x)$.

Also, for any arbitrary constant C, $\frac{d}{dx}[F(x)+C] = \frac{d}{dx}[F(x)] + 0 = f(x)$.

$\therefore \int f(x)dx = F(x) + C,$

This shows that F(x) and F(x) + C are both integrals of the same function f(x). Thus, for different values of C, we obtain different integrals of f(x). This implies that the integral of f(x) is not definite. By virtue of this property F(x) is called the indefinite integral of f(x).

1.1 Properties of Indefinite Integration

1. $\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$

2. $\int f'(x)dx = \int \frac{d}{dx}[f(x)]dx = f(x) + c$

3. $\int k\,f(x)dx = k\int f(x)dx$, where k is any constant

4. If $f_1(x), f_2(x), f_3(x)$, ... (finite in number) are functions of x, then

$$\int [f_1(x) \pm f_2(x) \pm f_3(x) ...]dx$$

$$= \int f_1(x)dx \pm \int f_2(x)dx \pm \int f_3(x)dx \pm ...$$

5. If $\int f(x)dx = F(x) + c$

then $\int f(ax \pm b)dx = \frac{1}{a}F(ax \pm b) + c$

1.2 Standard Formulae of Integration

The following results are a direct consequence of the definition of an integral.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1.$

2. $\int \frac{1}{x}dx = \log|x| + C$

3. $\int e^x\,dx = e^x + C$

4. $\int a^x\,dx = \frac{a^x}{\log_e a} + C.$

5. $\int \sin x\,dx = -\cos x + C$

6. $\int \cos x\,dx = \sin x + C$

7. $\int \sec^2 x\,dx = \tan x + C$

8. $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

9. $\int \sec x \tan x \, dx = \sec x + C$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C.$

11. $\int \tan x \, dx = -\log|\cos x| + C = \log|\sec x| + C.$

12. $\int \cot x \, dx = \log|\sin x| + C$

13. $\int \sec x \, dx = \log|\sec x + \tan x| + C$

14. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$

15. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C \; ; |x| < 1$

16. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

17. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x| + C \; ; |x| > 1$

2. METHODS OF INTEGRATION

2.1 Method of Substitution

By suitable substitution, the variable x in $\int f(x)\,dx$ is changed into another variable t so that the integrand $f(x)$ is changed into F(t) which is some standard integral or algebraic sum of standard integrals.

There is no general rule for finding a proper substitution and the best guide in this matter is experience.

However, the following suggestions will prove useful.

(i) If the integrand is of the form $f'(ax + b)$, then we put $ax + b = t$ and $dx = \frac{1}{a} dt$.

Thus, $\int f'(ax+b)\, dx = \int f'(t) \frac{dt}{a}$

$$= \frac{1}{a}\int f'(t)\, dt = \frac{f(t)}{a} = \frac{f(ax+b)}{a} + c$$

(ii) When the integrand is of the form $x^{n-1} f'(x^n)$, we put $x^n = t$ and $nx^{n-1} dx = dt$.

Thus, $\int x^{n-1} f'(x^n)\,dx = \int f'(t) \frac{dt}{n} = \frac{1}{n}$

$$\int f'(t)\,dt = \frac{1}{n} f(t) = \frac{1}{n} f(x^n) + c$$

(iii) When the integrand is of the form $[f(x)]^n \cdot f'(x)$, we put $f(x) = t$ and $f'(x)\,dx = dt$.

Thus, $\int [f(x)]^n f'(x)\,dx = \int t^n\,dt = \frac{t^{n+1}}{n+1} = \frac{[f(x)]^{n+1}}{n+1} + c$

(iv) When the integrand is of the form $\frac{f'(x)}{f(x)}$, we put $f(x) = t$ and $f'(x)\,dx = dt$.

Thus, $\int \frac{f'(x)}{f(x)} dx = \int \frac{dt}{t} = \log|t| = \log|f(x)| + c$

2.1.1 Some Special Integrals

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

3. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

4. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

5. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + C$

6. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

7. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

8. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

9. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

2.1.2 Integrals of the Form

(a) $\int f\left(a^2-x^2\right)dx,$

(b) $\int f\left(a^2+x^2\right)dx,$

(c) $\int f\left(x^2-a^2\right)dx,$

(d) $\int f\left(\frac{a-x}{a+x}\right)dx,$

Working Rule

Integral	**Substitution**
$\int f\left(a^2-x^2\right)dx,$	x = a sin θ or x = a cos θ
$\int f\left(a^2+x^2\right)dx,$	x = a tan θ or x = a cot θ
$\int f\left(x^2-a^2\right)dx,$	x = a sec θ or x = a cosec θ
$\int f\left(\frac{a-x}{a+x}\right)dx$ or $\int f\left(\frac{a+x}{a-x}\right)dx$	$x = a\cos 2\theta$

2.1.3 Integrals of the Form

(a) $\int \frac{dx}{ax^2+bx+c},$ (b) $\int \frac{dx}{\sqrt{ax^2+bx+c}},$

(c) $\int \sqrt{ax^2+bx+c}\,dx$ a,

Working Role

(i) Make the coefficient of x^2 unity by taking the coefficient of x^2 outside the quadratic.

(ii) Complete the square in the terms involving x, i.e. write $ax^2 + bx + c$ in the form

$$a\left[\left(x+\frac{b}{2a}\right)^2\right] - \frac{\left(b^2-4ac\right)}{4a}.$$

(iii) The integrand is converted to one of the nine special integrals.

(iv) Integrate the function.

2.1.4 Integrals of the Form

(a) $\int \frac{px+q}{ax^2+bx+c}dx,$ (b) $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx,$

(c) $\int (px+q)\sqrt{ax^2+bx+c}\;dx$

Integral Working Rule

$\int \frac{px+q}{ax^2+bx+c}dx$ Put $px + q = \lambda\,(2ax + b) + \mu$ or $px + q = \lambda$ (derivative of quadratic) + μ.

Comparing the coefficient of x and constant term on both sides, we get

$p = 2a\lambda$ and $q = b\lambda + \mu \Rightarrow \lambda = \frac{}{2a}$ and $\mu = \left(q - \frac{bp}{2a}\right)$. Then integral becomes

$$\int \frac{px+q}{ax^2+bx+c}dx$$

$$= \frac{}{2a}\int \frac{2ax+b}{ax^2+bx+c}dx + \left(q-\frac{bp}{2a}\right)\int \frac{dx}{ax^2+bx+c}$$

$$= \frac{}{2a}\log|ax^2+bx+c| + \left(q-\frac{bp}{2a}\right)\int \frac{dx}{ax^2+bx+c}$$

$\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$ In this case the integral becomes

$$\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx =$$

$$\frac{p}{2a}\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}\,dx + \left(q-\frac{bp}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$= \frac{p}{a}\sqrt{ax^2+bx+c} + \left(q-\frac{bp}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int (px+q)\sqrt{ax^2+bx+c}\,dx$$

The integral in this case is converted to

$$\int (px+q)\sqrt{ax^2+bx+c}\,dx = \frac{p}{2a}\int (2ax+b)\sqrt{ax^2+bx+c}\,dx$$

$$+\left(q-\frac{bp}{2a}\right)\int \sqrt{ax^2+bx+c}\,dx$$

$$= \frac{p}{3a}(ax^2+bx+c)^{3/2} + \left(q-\frac{bp}{2a}\right)\int \sqrt{ax^2+bx+c}\,dx$$

2.1.5 Integrals of the Form

$\int \frac{P(x)}{\sqrt{ax^2+bx+c}}\,dx$, where $P(x)$ is a polynomial in x of degree $n \geq 2$.

Working Rule: Write

$$\int \frac{P(x)}{\sqrt{ax^2+bx+c}}\,dx =$$

$$= (a_0 + a_1x + a_2x^2 + ... + a_{n-1}x^{n-1})\ \sqrt{ax^2+bx+c} + k\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

where k, a_0, a_1, ... a_{n-1} are constants to be determined by differentiating the above relation and equating the coefficients of various powers of x on both sides.

2.1.6 Integrals of the Form

$$\int \frac{x^2+1}{x^4+kx^2+1}\,dx \quad \text{or} \quad \int \frac{x^2-1}{x^4+kx^2+1}\,dx,$$

where k is a constant positive, negative or zero.

Working Rule

(i) Divide the numerator and denominator by x^2.

(ii) Put $x-\frac{1}{x}=z$ or $x+\frac{1}{x}=z$, whichever subsitution, on differentiation gives, the numerator of the resulting integrand.

(iii) Evaluate the resulting integral in z

(iv) Express the result in terms of x.

2.1.7 Integrals of the Form

$\int \frac{dx}{P\sqrt{Q}}$, where P, Q are linear or quadratic functions of x.

Integral	**Substitution**
$\int \frac{1}{(ax+b)\sqrt{cx+d}}\,dx$	$cx + d = z^2$
$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$	$px + q = z^2$
$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$	$px + q = \frac{1}{z}$
$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$	$x = \frac{1}{z}$

3. METHOD OF PARTIAL FRACTIONS FOR RATIONAL FUNCTIONS

Integrals of the type $\int \frac{p(x)}{g(x)}$ can be integrated by resolving the integrand into partial fractions. We proceed as follows:

Check degree of p (x) and g (x).

If degree of p (x) $\geq$ degree of g (x), then divide p (x) by g (x) till its degree is less, i.e. put in the form $\frac{p(x)}{g(x)} = r(x) + \frac{f(x)}{g(x)}$ where degree of f (x) < degree of g (x).

CASE 1: When the denominator contains non-repeated linear factors. That is

$$g(x) = (x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n).$$

In such a case write f (x) and g (x) as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{(x-\alpha_1)} + \frac{A_2}{(x-\alpha_2)} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where A_1, A_2, ... A_n are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 2 : When the denominator contains repeated as well as non-repeated linear factor. That is

$$g(x) = (x - \alpha_1)^2 (x - \alpha_3) \ldots (x - \alpha_n).$$

In such a case write f (x) and g (x) as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{x-\alpha_1} + \frac{A_2}{(x-\alpha_1)^2} + \frac{A_3}{x-\alpha_3} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where A_1, A_2, ... A_n are constants to determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

Note : Corresponding to repeated linear factor $(x - a)^r$ in the denominator, a sum of r partial fractions of the type $\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_r}{(x-a)^r}$ is taken.

CASE 3 : When the denominator contains a non repeated quadratic factor which cannot be factorised further:

$$g(x) = (ax^2 + bx + c)(x - \alpha_3)(x - \alpha_4) \ldots (x - \alpha_n).$$

In such a case express f (x) and g (x) as:

$$\frac{f(x)}{g(x)} = \frac{A_1x + A_2}{ax^2 + bx + c} + \frac{A_3}{x-\alpha_3} + \ldots + \frac{A_n}{x-\alpha_n}$$

where A_1, A_2, ... A_n are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 4 : When the denominator contains a repeated quadratic factor which cannot be factorised further: That is

$$g(x) = (ax^2 + bx + c)^2 (x - \alpha_5)(x - \alpha_6) \ldots (x - \alpha_n)$$

In such a case write f (x) and g (x) as

$$\frac{f(x)}{g(x)} = \frac{A_1x + A_2}{ax^2 + bx + c} + \frac{A_3x + A_4}{(ax^2 + bx + c)^2} + \frac{A_5}{x-\alpha_5} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where A_1, A_2, ... A_n are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 5 : If the integrand contains only even powers of x

(i) Put $x^2 = z$ in the integrand.

(ii) Resolve the resulting rational expression in z into partial fractions

(iii) Put $z = x^2$ again in the partial fractions and then integrate both sides.

4. METHOD OF INTEGRATION BY PARTS

The process of integration of the product of two functions is known as integration by parts.

For example, if u and v are two functions of x, then $\int (uv)\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx \right) dx.$

In words, integral of the product of two functions = first function × integral of the second – integral of (differential of first × integral of the second function).

Working Hints

(i) Choose the first and second function in such a way that derivative of the first function and the integral of the second function can be easily found.

(ii) In case of integrals of the form $\int f(x).x^n$ dx, take x^n as the first function and $f(x)$ as the second function.

(iii) In case of integrals of the form $\int (\log x)^n \cdot 1 dx$, take 1 as the second function and $(\log x)^n$ as the first function.

(iv) Rule of integration by parts may be used repeatedly, if required.

(v) If the two functions are of different type, we can choose the first function as the one whose initial comes first in the word "ILATE", where

I — Inverse Trigonometric function

L — Logarithmic function

A — Algebraic function

T — Trigonometric function

E — Exponential function.

(vi) In case, both the functions are trigonometric, take that function as second function whose integral is simple. If both the functions are algebraic, take that function as first function whose derivative is simpler.

(vii) If the integral consists of an inverse trigonometric function of an algebraic expression in x, first simplify the integrand by a suitable trigonometric substitution and then integrate the new integrand.

4.1 Integrals of the Form

$$\int e^x\left[f(x)+f'(x)\right]dx$$

Working Rule

(i) Split the integral into two integrals.

(ii) Integrate only the first integral by parts, i.e.

$$\int e^x\left[f(x)+f'(x)\right] x = \int e^x f(x) \quad x + \int e^x f'(x)\,dx$$

$$= \left[f(x).e^x - \int f'(x).e^x \quad x\right] + \int e^x f'(x)\,dx$$

$$= e^x f(x) + C.$$

4.2 Integrals of the Form:

Where the initial integrand reappears after integrating by parts.

Working Rule

(i) Apply the method of integration by parts twice.

(ii) On integrating by parts second time, we will obtain the given integrand again, put it equal to I.

(iii) Transpose and collect terms involving I on one side and evaluate I.

5. INTEGRAL OF THE FORM (TRIGONOMETRIC FORMATS)

5.1 (a) $\int \frac{dx}{a+b\cos x}$ (b) $\int \frac{dx}{a+b\sin x}$

(c) $\int \frac{dx}{a+b\cos x+c\sin x}$

Working Rule

(i) Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ so that the given integrand becomes a function of $\tan\frac{x}{2}$.

(ii) Put $\tan\frac{x}{2} = z \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

(iii) Integrate the resulting rational algebraic function of z

(iv) In the answer, put $z = \tan\frac{x}{2}$.

5.2 Integrals of the Form

(a) $\int \frac{dx}{a+b\cos^2 x}$ (b) $\int \frac{dx}{a+b\sin^2 x}$

(c) $\int \frac{dx}{a\cos^2 x + b\sin x\cos x + c\sin^2 x}$

Working Rule

(i) Divide the numerator and denominator by $\cos^2 x$.

(ii) In the denominator, replace $\sec^2 x$, if any, by $1 + \tan^2 x$.

(iii) Put $\tan x = z \Rightarrow \sec^2 x\, dx = dz$.

(iv) Integrate the resulting rational algebraic function of z.

(v) In the answer, put z = tan x.

5.3 Integrals of the Form

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\, dx$$

Working Role

(i) Put Numerator = λ (denominator) + μ (derivative of denominator)

$a\cos x + b\sin x = \lambda(c\cos x + d\sin x) + \mu(-c\sin x + d\cos x)$.

(ii) Equate coefficients of sin x and cos x on both sides and find the values of λ and μ.

(iii) Split the given integral into two integrals and evaluate each integral separately, i.e.

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\, dx =$$

$$\lambda \int 1dx + \mu \int \frac{-c\sin x + d\cos x}{a\cos x + b\sin x}\, dx = \lambda x + \mu \log|a\cos x + b\sin x|.$$

(iv) Substitute the values of λ and μ found in step 2.

5.4 Integrals of the Form

$$\int \frac{a + b\cos x + c\sin x}{e + f\cos x + g\sin x}\, dx$$

Working Rule

(i) Put Numerator = l (denominator) + m (derivative of denominator) + n

$a + b\cos x + c\sin x = l(e + f\cos x + g\sin x) + m(-f\sin x + g\cos x) + n$

(ii) Equate coefficients of sin x, cos x and constant term on both sides and find the values of l, m, n.

(iii) Split the given integral into three integrals and evaluate each integral separately, i.e.

$$\int \frac{a + b\cos x + c\sin x}{e + f\cos x + g\sin x}\, dx$$

$$l \int 1dx + m \int \frac{-f\sin x + g\cos x}{e + f\cos x + g\sin x}\, dx +$$

$$n \int \frac{dx}{e + f\cos x + g\sin x}$$

$$= lx + m\log|e + f\cos x + g\sin x| + n\int \frac{dx}{e + f\cos x + g\sin x}\, dx$$

(iv) Substitute the values of l, m, n found in Step (ii).

5.5 Integrals of the Form

$$\int \sin^m x \cos^n x\, dx$$

Working Rule

(i) If the power of sin x is an odd positive integer, put cos x = t.

(ii) If the power of cos x is an odd positive integer, put sin x = t.

(iii) If the power of sin x and cos x are both odd positive integers, put sin x = t or cos x = t.

(iv) If the power of sin x and cos x are both even positive integers, use De' Moivre's theorem as follows:

Let, $\cos x + i\sin x = z$. Then $\cos x - i\sin x = z^{-1}$

Adding these, we get $z + \frac{1}{z} = 2\cos x$ and $z - \frac{1}{z} = 2i\sin x$

By De'Moivre's theorem, we have

$z^n + \frac{1}{z^n} = 2\cos nx$ and $z^n - \frac{1}{z^n} = 2i\sin^n x$ ...(1)

$$\therefore \quad \sin^m x \cos^n x = \frac{1}{(2i)^m}\cdot\frac{1}{2^n}\left(z + \frac{1}{z}\right)^n\left(z - \frac{1}{z}\right)^m$$

$$= \frac{1}{2^{m+n}}\cdot\frac{1}{i^m}\left(z + \frac{1}{z}\right)^n\left(z - \frac{1}{z}\right)^m.$$

Now expand each of the factors on the R.H.S. using Binomial theorm. Then group the terms equidistant from the beginning and the end. Thus express all such pairs as the sines or cosines of multiple angles. Further integrate term by term.

(v) If the sum of powers of sin x and cos x is an even negative integer, put tan x = z.

SOLVED EXAMPLES

Example – 1

Evaluate : $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

Sol. $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

$= \int x^3\, dx + \int 5x^2\, dx - \int 4dx + \int \frac{7}{x} dx + \int \frac{2}{\sqrt{x}} dx$

$= \int x^3\, dx + 5 . \int x^2\, dx - 4 . \int 1 . dx + 7 . \int \frac{1}{x} dx + 2 . \int x^{-1/2} dx$

$= \frac{x^4}{4} + 5 . \frac{x^3}{3} - 4x + 7 \log|x| + 2\left(\frac{x^{1/2}}{1/2} \right) + C$

$= \frac{x^4}{4} + \frac{5}{3} x^3 - 4x + 7 \log|x| + 4\sqrt{x} + C$

Example – 2

Evaluate : $\int e^{x \log a} + e^{a \log x} + e^{a \log a}\, dx$

Sol. We have,

$\int e^{x \log a} + e^{a \log x} + e^{a \log a}\, dx$

$= \int e^{\log a^x} + e^{\log x^a} + e^{\log a^a}\ dx$

$= \int (a^x + x^a + a^a)\, dx$

$= \int a^x\, dx + \int x^a\, dx + \int a^a\, dx$

$= \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a . x + C.$

Example – 3

Evaluate : $\int \frac{x^4}{x^2 + 1}\ dx$

Sol. $\int \frac{x^4}{x^2 + 1}\ dx$

$= \int \frac{x^4 - 1 + 1}{x^2 + 1}\ dx = \int \frac{x^4 - 1}{x^2 + 1} + \frac{1}{x^2 + 1}\ dx$

$= \int (x^2 - 1)\ dx + \int \frac{1}{x^2 + 1}\ dx = \frac{x^3}{3} - x + \tan^{-1} x + C$

Example – 4

Evaluate : $\int \frac{2^x + 3^x}{5^x}\ dx$

Sol. $\int \frac{2^x + 3^x}{5^x}\ dx$

$= \int \left(\frac{2^x}{5^x} + \frac{3^x}{5^x} \right) dx$

$= \int \left[\left(\frac{2}{5} \right)^x + \left(\frac{3}{5} \right)^x \right] dx = \frac{(2/5)^x}{\log_e 2/5} + \frac{(3/5)^x}{\log_e 3/5} + C$

Example – 5

Evaluate : $\int x^3 \sin\ ^{4}\ dx$

Sol. We have

$I = \int x^3 \sin\ ^{4}\ dx$

Let $x^4 = t \Rightarrow d(x^4) = dt$

$\Rightarrow 4x^3\, dx = dt \Rightarrow dx = \frac{1}{4x^3}\ dt$